CSE 140 Notes

- **Basic**
  - 0.0=0 | 0+0=0
  - 1.1=1 | 1+1=1
  - 1.0=0 | 1+0=1
- **Identity laws**
  - X.0=0 | X+0=X
  - X.1=X | X+1=1
- **Complement laws**
  - X.X'=0 | X+X'=1
- **Commutative laws**
  - X.Y=Y.X | X+Y=Y+X
- **Distributive laws**
  - X.(Y+Z)=X.Y+X.Z | X+(YZ)=(X+Y).(X+Z)
- **Associative laws**
  - X.(Y.Z)=(X.Y).Z | X+(Y+Z)=(X+Y)+Z
- **Consensus theorem**
  - XY+Y'Z+XZ = XY+Y'Z | (X+Y).(Y'+Z).(X+Z)=(X+Y)(Y'+Z)
- **DeMorgan's theorem**
  - (X.Y)' = X' + Y' | (X+Y)'=X'.Y'
- **Shannon's expansion**
  - f(x,y)=x.f(1,y)+x'.f(0,y) | f(x,y)=[x+f(0,y)] . [x'+f(1,y)]
- **Sum of products -** *Minterms*
  - Look at the rows with output 1
  - Try to force the inputs to be 1's
- **Product of sums -** *Maxterms*
  - Look at the rows with output 0
  - Try to force the inputs to be 0's
- **Kmap**
  - Adjacent squares
  - Prime Implicants
  - Essential Implicants
  - *Circle biggest adjacent squares with size = power of 2*

Dao Lam

Boolean Algebra

Exercise 1: Prove ab+b'cd+acd=ab+b'cd

Step 1: Look at the *term that has complement*, which is b in this case.

Step 2: Pick either b+b' or b.b' to *insert into the extra term*, which is acd (left side has it and right side doesn't). Remember to use the same operator, in this case, it's AND.

It would either be acd(b+b') or acd(b.b'). Which one to use?

acd(b.b') = acd(0) = 0! We cannot use this since inserting b.b' changes acd to 0!

acd(b+b') = acd(1) = acd. This is good, since inserting (b+b') does not affect acd.

Step 3: Change the LHS to ab+b'cd+acd(b+b')

Step 4: Use distributive laws & identity laws

LHS	= ab+b'cd+acd(b+b')	
	= ab+b'cd+abcd+ab'cd	//distributive laws
	= ab(1+cd)+b'cd(1+a)	//distributive laws
	= ab+b'cd	//identity laws
	= RHS	

Exercise 2: Prove (a+b)(b'+c+d)(a+c+d) = (a+b)(b'+c+d)

Step 1: Look at the *term that has complement*, which is b in this case.

Step 2: Pick either b+b' or b.b' to *insert into the extra term*, which is a+c+d (left side has it and right side doesn't). Remember to use the same operator. In this case, it's OR.

It would either be a+c+d+(b+b') or a+c+d+(b.b'). Which one to use?

a+c+d+bb' = a+c+d+0 = a+c+d This is good since inserting it does not change a+c+d.

a+c+d+(b+b') = a+c+d+1 = 1 This is bad since (a+c+d) just changed to 1!

Step 3: Change the LHS to (a+b)(b'+c+d)(a+c+d+bb')

Step 4: Use distributive laws & identity laws

LHS	= (a+b)(b'+c+d)(a+c+d+bb')	
	= (a+b)(b'+c+d)(a+b+c+d)(a+b'+c+d)	//distributive laws
	= (a+b)(1+c+d)(b'+c+d)(1+a)	//distributive laws
	= (a+b)(b'+c+d)	//identity laws
	= RHS	

Shannon's Expansion

Exercise 1: Prove ab+b'cd+acd=ab+b'cd

Step 1: Look at the *term that has complement*, which is b in this case.

Step 2: Let LHS = f(a,b,c,d) = ab+b'cd+acd

Step 3: Replace b with 0 and 1, compute f(a,0,c,d) and f(a,1,c,d). Use distributive laws & identity laws to simplify it as much as possible.

f(a,0,c,d) = a0 + 1cd + acd = 0 + cd + acd = cd + acd = cd(1+a) = cd

f(a,1,c,d) = a1 + 0cd + acd = a + 0 + acd = a + acd = a(1+cd) = a

Step 4: Apply Shannon's Expansion

LHS = b.f(a,1,c,d) + b'.f(a,0,c,d)

Step 5: Plug f(a,1,c,d) and f(a,0,c,d) into the above expression. Again, use distributive laws & identity laws to simplify it.

LHS = b.f(a,1,c,d) + b'.f(a,0,c,d)
= ba+ b'cd
= ab + b'cd //commutative laws
= RHS

Exercise 2: Prove (a+b)(b'+c+d)(a+c+d) = (a+b)(b'+c+d)

Step 1: Look at the *term that has complement*, which is b in this case.

Step 2: Let LHS = f(a,b,c,d) = (a+b)(b'+c+d)(a+c+d)

Step 3: Replace b with 1 and 0, compute f(a,0,c,d) and f(a,1,c,d). Use distributive laws & identity laws to simplify it as much as possible.

f(a,0,c,d) = (a+0)(1+c+d)(a+c+d) = a(a+c+d) = a(1+c+d) = a

f(a,1,c,d) = (a+1)(0+c+d)(a+c+d) = (c+d)(a+c+d) = (c+d)(1+a) = c+d

Step 4: Apply Shannon's Expansion

LHS = [b+f(a,0,c,d)] . [b'+f(a,1,c,d)]

Step 5: Plug f(a,0,c,d) and f(a,1,c,d) into the above expression. Again, use distributive laws & identity laws to simplify it.

LHS = [b+f(a,0,c,d)] . [b'+f(a,1,c,d)]
= (b+a) (b'+c+d)
= (a+b) (b'+c+d) //commutative laws
= RHS